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RYDBERG CONSTANT AND THE GOLDEN SECTION

Abstract:

It is found in the Bohr`s theory of hydrogen atom that there are the connections of the fundamental math constants , the golden section (phi) and ratio of circumference to its diameter (π) , with the fundamental constants of atom: Rydberg constant (R) , Fine structure constant (α), the electron charge (e), the Planck constant (h), electron rest mass (m), radius of electron`s orbit (r), velocity of electron in atom (v) and the speed of light (c).

Johaness Robert Rydberg was a Swedish physicist noted for his work in spectroscopy.The son of a merchant and ship owner, Rydberg was born in Halmstad. He enrolled in the University of Lund in 1873,receiving his bachelor's degree in mathematics in 1875 and his doctorate in 1879 for his thesis on the construction of conic sections.In 1882 Rydberg moved from a lectureship in mathematics to become a lecturer in physics at Lund.

Ten years later he was promoted to assistant at the Physics Institute. In 1879 he was promoted to a professorship in physics but the was only a temporary position until it was confirmed as a permanent appointment in March 1901. From this time until his retirement in 1919 he held the chair of physics at Lund.

However, his health deteriotated during the time that he held the chair and he became seriously ill in 1914. Although he continued to hold the chair he took sick leave in 1914 and was absent from the university from that time on. His final retirement came five years after he had ceased to be able to work and came only a few weeks before his death.
Rydberg is best known for his theoretical studies of spectral series. Using wave numbers instead of wavelengths in his calculations, he was able to arrive at a relatively simple expression that related the various lines in the spectra of chemical elements. The expression contained a constant term that became known as the Rydberg constant.

The Rydberg constant, R, represents the limiting value of the highest wavenumber (the inverse wavelength) of any photon that can be emitted from the hydrogen atom, or, alternatively, the wavenumber of the lowest-energy photon capable of ionizing the hydrogen atom from its ground state. The spectrum of hydrogen can be expressed simply in terms of the Rydberg constant, using the Rydberg formula.

Rydberg constant is often used in atomic physics in the form of an energy.

$$hcR = E_{kinetic}$$

Or

$$hcR = \frac{mV_{Bohr}{}^2}{2}$$

Which gives

$$R = \frac{mc\alpha^2}{2h}$$

Or

$$R = \frac{\alpha^2}{2\lambda_{Compton}}$$

As we know

$$\alpha^2 = 16\pi^2 r_{Bohr}{}^2 R^2 = 4\lambda_{Bohr}{}^2 R^2$$

Then we get

$$2\lambda_{Compton} = 4R\lambda_{Bohr}{}^{2}$$

Or finally:

$$R = \frac{\lambda_{Compton}}{2\lambda_{Bohr}{}^{2}}$$

The Rydberg constant can also be expressed in terms of the Golden Section.

According to the Bohr`s theory of the hydrogen atom, the electron is held in a circular orbit by electrostatic attraction. The centripetal force is equal to the Coulomb force.

$$\frac{mV_{Bohr}{}^{2}}{r_{Bohr}} = \frac{e^2c^2 10^{-7}}{r_{Bohr}{}^{2}}$$

$$mV_{Bohr}{}^{2} = \frac{e^2c^2 10^{-7}}{r_{Bohr}{}^{2}}$$

$$mc^2\alpha^2 = \frac{e^2c^2 10^{-7}}{r_{Bohr}}$$

$$1 = \frac{e^2 10^{-7}}{(4\pi r_{Bohr})^2 R^2 (mr_{Bohr})}$$

$$R^2 = \frac{e^2 10^{-7}}{2^2 (2\pi r_{Bohr})^2 (mr_{Bohr})}$$

finally

$$R = \frac{e}{2\lambda_{Bohr}} \sqrt{\frac{1}{10mr_{Bohr}}} * 10^{-3}$$

Or

$$R = \frac{\alpha}{4\pi r_{Bohr}}$$

The Compton wavelength is a quantum mechanical property of a particle. The Compton wavelength of a particle is equivalent to the wavelength of a photon whose energy is the same as the rest-mass energy of the particle. For the Compton wavelength we have:

$$\lambda_{Compton} = 2R\lambda_{Bohr}{}^{2}$$

$$\lambda_{Compton} = 2\lambda_{Bohr}{}^{2}\frac{e*10^{-3}}{2\lambda_{Bohr}}\sqrt{\frac{1}{10mr_{Bohr}}}$$

finally

$$\lambda_{Compton} = \lambda_{Bohr}e\sqrt{\frac{1}{10mr_{Bohr}}}*10^{-3}$$

or

$$\lambda_{Compton} = \alpha\lambda_{Bohr}$$

Where

$$\sqrt{10} = \sqrt{2}\sqrt{5} = (Phi + phi)\sqrt{2}$$

Phi and phi are The Golden section constants. And where :

$e - elementary\_charge$

$h - Planck\_constant$

$m - electron\_rest\_mass$

$c - speed\_of\_light$

$V_{Bohr} - Bohr`s\_velocity\_of\_electron$

$r_{Bohr} - Bohr`s\_radius$

$\lambda_{Bohr} - Bohr`s\_wavelength$

$\lambda_{Compton} - Compton\_wavelength$

$\alpha - Fine\_structure\_constant = 7.29...10^{-3}$

$R - Rydberg\_constant(R_{\infty})$

$Phi - The\_golden\_section\_constant = 1.618...$

$phi - the\_golden\_section\_constant = 0.618...$

$\pi - number\_PI = 3.14...$

According to 2006 CODATA results:

R=10 973 731.568 527(72) m^{-1}.