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DIRAC`S CONSTANT AND THE GOLDEN SECTION

Abstract:

It is found in the Bohr`s theory of hydrogen atom that there are the connections of the fundamental math constants , the golden section (phi, Phi) and ratio of circumference to its diameter (π) , with the fundamental constants of atom: Rydberg constant (R) , Fine structure constant (α), the electron charge (e), the Planck constant (h), the Dirac`s constant ( $\hbar$ ), electron rest mass ( m ), radius of electron`s orbit (r_{Bohr}), velocity of electron in atom (V_{Bohr}) and the speed of light (c).

In physics, action is an attribute of the physical system dynamics. It has units of energy x time (joule – seconds).
Planck`s constant is the quantum of action.
According to the quantum theory of hydrogen atom, Planck`s constant is:

$$h = E_{potential} T_{Bohr}$$

And

$$h = F_{orbit} \lambda_{Bohr} T_{Bohr}$$

So we have this physical constant

$$h = E_{potential} T_{Bohr}$$
$$h = m V_{Bohr}^{2} T_{Bohr}$$
$$h = m V_{Bohr} (V_{Bohr} T_{Bohr})$$
$$h = m V_{Bohr} \lambda_{Bohr}$$
$$h = 2\pi * m V_{Bohr} r_{Bohr}$$

Niels Bohr applies quantum theory to the Rutherford`s atomic structure by assuming that electrons travel in stationary orbits by their angular momentum. According to the Bohr the angular momentum in hydrogen atom is

$$L = m V_{Bohr} r_{Bohr} = \hbar$$

Where $\hbar$ is Dirac`s constant

$$\hbar = \frac{h}{2\pi}$$

Dirac`s constant used in quantum mechanics, equal to the Planck`s constant divided by 2π .
Also called crossed - h, h bar.
Named after Paul Adrien Maurice Dirac (1902 – 1984) , English physicist.
Planck`s constant, as the action, equals:

$$h = F_{orbit}\lambda_{Bohr}T_{Bohr}$$

$$F_{orbit} = \frac{mV_{Bohr}^{2}}{\lambda_{Bohr}}$$

$$h = \frac{mV_{Bohr}^{2}}{\lambda_{Bohr}}\lambda_{Bohr}T_{Bohr}$$

$$h = mV_{Bohr}(V_{Bohr}T_{Bohr})$$

$$h = mV_{Bohr}\lambda_{Bohr}$$

Or

$$\lambda_{Bohr} = \frac{h}{mV_{Bohr}}$$

This result of Bohr`s hydrogen atom theory (special case) , become the de Broglie relation (general case). For the hydrogen atom the de Broglie relations are :

$$\lambda_{Bohr} = \frac{h}{mV_{Bohr}}$$

$$\nu_{Bohr} = \frac{E_{potential}}{h}$$

In an electromagnetic fields the wave - like behavior of small – momentum particles is analogous to that of light.
On the other hand, in a gravitational fields the particles – like behavior of small – wave`s energy is analogous to that of electron.

The de Broglie equations relate the wavelength λ and frequency ν to the momentum p And energy E , respectively as

$$p = \hbar k$$

$$E = \hbar\omega$$

Where $\hbar$ is the reduced Planck`s constant ( Dirac`s constant) , k is the angular wave number and ω is the angular frequency.

Also we have Planck`s constant definition as:

$$h = F_{orbit}\lambda_{Bohr}T_{Bohr}$$

$$F_{orbit} = \frac{e^2c^2 10^{-7}}{\lambda_{Bohr}r_{Bohr}}$$

$$h = 2\pi\frac{e^2c^2 10^{-7}}{\lambda_{Bohr}^{\ 2}}\frac{\lambda_{Bohr}}{\nu_{Bohr}}$$

$$h = 2\pi\frac{e^2c^2 10^{-7}}{h\nu_{Bohr}}mV_{Bohr}$$

$$h\nu_{Bohr} = mV_{Bohr}^{\ 2}$$

$$h = 2\pi\frac{e^2c^2 10^{-7}}{mV_{Bohr}^{\ 2}}mV_{Bohr}$$

$$h = 2\pi\frac{e^2c^2 10^{-7}}{V_{Bohr}}$$

$$V_{Bohr} = c\alpha$$

$$h = 2\pi\frac{e^2c\ 10^{-7}}{\alpha}$$

$$\hbar = \frac{e^2c}{\alpha} * 10^{-7}$$

The relation between fine structure constant and the golden section is:

$$\alpha = \frac{e}{\sqrt{10mr_{Bohr}}} * 10^{-3}$$

The connection between Dirac`s constant and the Golden section is

$$\hbar = ec\sqrt{\frac{mr_{Bohr}}{10}} * 10^{-3}$$

On the other hand

$$E_{potential} = mV_{Bohr}^{\ 2}$$

$$V_{Bohr} = \frac{h}{m\lambda_{Bohr}}$$

$$mV_{Bohr}{}^{2} = \frac{h^{2}}{m\lambda_{Bohr}{}^{2}}$$

$$mV_{Bohr}{}^{2} = \frac{e^{2}c^{2}10^{-7}}{r_{Bohr}}$$

$$\lambda_{Bohr} = 2\pi r_{Bohr}$$

$$h^{2} = (2\pi ec)^{2}(\frac{mr_{Bohr}}{10}) * 10^{-6}$$

$$h = 2\pi ec\sqrt{\frac{mr_{Bohr}}{10}} * 10^{-3}$$

This is the connection between Dirac`s constant and the value of the Golden Section

$$\hbar = ec\sqrt{\frac{mr_{Bohr}}{10}} * 10^{-3}$$

Where

$$\sqrt{10} = \sqrt{2} * (Phi + phi)$$

Phi and phi are values of the Golden section.